Physical constraints on c_{13} and Thomsen parameter delta for VTI rocks

Fuyong Yan, De-hua Han and Qiuliang Yao, Rock Physics Lab, University of Houston*

Summary

From observation of static mechanic measurement of VTI rock or rock-like VTI materials, we reasoned that one of the three primary Poisson's ratios of real VTI rocks should always be bigger than the other two and they should be generally positive. From these relations we derived strict physical constraints on c_{13} and Thomsen parameter δ. Some of the published data from lab velocity anisotropy measurement are lying outside of the constraints, we analyzed that it is primarily caused by the big uncertainty associated with the diagonal phase velocity measurement. These physical constraints will be useful for our understanding of Thomsen parameter δ and predicting δ from non-diagonal measurement.

Introduction

Thomsen (1986) defined a set of parameters (ε, γ and δ) and brought up weak anisotropy approximation for phase velocities in the VTI medium. These parameters and the linearized approximation are widely accepted and used in the industry. With increasing importance of organic shale as a reservoir rock, laboratory measurements on velocity anisotropy of core plugs are done routinely. The results are usually reported in terms of Thomsen parameters (Vernik and Nur, 1992; Johnston,1995; Vernik and Liu, 1997; Jakobsen and Johansen, 2000; Wang, 2002; Sondergeld and Rai, 2011 and Sone, 2012). Of the three parameters, δ is one of the most important parameters for exploration geophysicist since it determines the NMO behavior of VTI layers (how it differs from the isotropic case). As Thomsen (1986) pointed out himself that δ is an "awkward" parameter and its physical meaning is not straight forward. In spite of large amount of lab measurement, our understanding of the parameter δ is still not quite clear. The lab measurement found that δ has very poor correlation with other Thomsen parameters, and even the rational data range of δ is not certain.

Of the five independent elastic constants (c_{11}, c_{33}, c_{44}, c_{66} and c_{13}) of a VTI medium, although mathematically they are free independent variables, good to excellent mutual correlations are found between c_{11}, c_{33}, c_{44}, and c_{66} for natural rocks from lab velocity anisotropy measurement. But behavior of c_{13} is erratic, it seems that no correlations exist between c_{13} other elastic constants. We believe that for natural VTI rocks, there should exists some form of constrains on c_{13}. If we know behavior of c_{13}, then we can have better understanding of Thomsen parameter δ.

Theory

Young's modulus and Poisson's ratio are basic parameters to describe material mechanical properties. For isotropic medium, from the definitions and using Hook's law, they are related to elastic constants as follows(Marvko, et. al.,1998):

$$E=\frac{9K\mu}{3K+\mu}, \quad \nu=\frac{3K-2\mu}{2(3K+\mu)} \tag{1}$$

The theoretical value of ν lies between [-1, 0.5](Thomsen, 1990; Carcione, 2002). The Poisson's ratio of foam and some network materials can be negative(Greaves, et. al. 2011). For natural isotropic rock, a practical limits of Poison's ration is $0<\nu<0.5$ (Gercek, 2007).

The concepts of Young's modulus and Poisson's ratio can be straightforwardly extended to VTI medium using Hook's law(King, 1964; Banik, 2011). Their relations with the elastic constants are as follows:

$$E_V=\frac{c_{33}(c_{11}-c_{66})-c_{13}^2}{c_{11}-c_{66}} \qquad (=E_3) \tag{2}$$

$$E_H=\frac{4c_{66}(c_{33}(c_{11}-c_{66})-c_{13}^2)}{c_{11}c_{33}-c_{13}^2} \qquad (=E_1=E_2) \tag{3}$$

$$\nu_V=\frac{c_{13}}{2(c_{11}-c_{66})} \qquad (=\nu_{31}=\nu_{32}) \tag{4}$$

$$\nu_{HV}=\frac{2c_{13}c_{66}}{c_{11}c_{33}-c_{13}^2} \qquad (=\nu_{13}=\nu_{23}) \tag{5}$$

$$\nu_{HH}=\frac{c_{33}(c_{11}-2c_{66})-c_{13}^2}{c_{11}c_{33}-c_{13}^2} \qquad (=\nu_{12}=\nu_{21}) \tag{6}$$

The coordinate system used for the notation is shown in

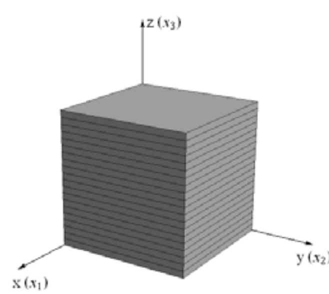

Figure 1: Right-handed coordinate system used in this study

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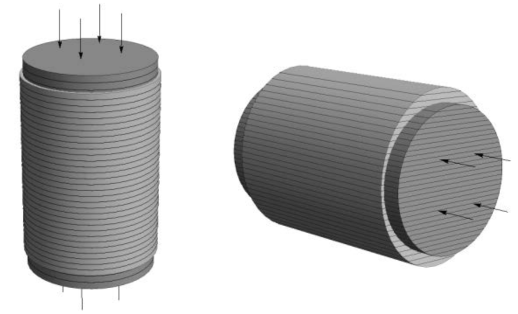

Figure 2 Schematic deformation of vertical plug(left) and horizontal plug(right) of VTI rocks under axial compression testing

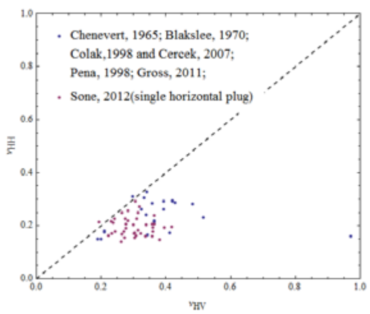

Figure 3: Static mechanic measurement of poisson's ratios on VTI rocks and rock-like materials.

Fig. 1. An important relation exists between ν_V and ν_{HV}:

$$\nu_{HV} = \frac{E_H}{E_V}\nu_V \tag{7}$$

For VTI medium, the Young's modulus in horizontal direction should always be bigger than that in vertical direction (E_H>E_V), so that $\nu_{HV} > \nu_V$.

Physical constraints on c_{13} and δ

Fig. 2 shows the schematic deformation of vertical plug and horizontal plug of VTI rocks under axial compression test. In the left panel, the transverse deformation is identical in every direction, it is physically intuitional that the plug will not shrink transversely under axial compression, thus there is only one Poisson's ratio (ν_V) and it is positive, thus from eqn. (4) and c_{11}>c_{66} for VTI medium, we get

$$c_{13} > 0 \tag{8}$$

In the right panel of Fig. 2, when a horizontal plug is under uniform axial stress, the transverse deformation will not be uniform. There are two principal Poisson's ratios: ν_{HH} and ν_{HV}. Since VTI rocks is harder in horizontal direction than in vertical direction(E_H>E_V), when under axial compression, the rock is more resistant to deformation (expansion) in horizontal direction than in vertical direction. There should be no shrinkage in transverse directions. Thus we have:

$$0 < \nu_{HH} < \nu_{HV} \tag{9}$$

This relation is validated by laboratory static mechanic measurement as shown in Fig. 3. The two data points showing ν_{HH} bigger than ν_{HV} might be caused to measurement uncertainty or the material are not real VTI media.

Thus ν_{HV} has highest value among the three primary Poisson's ratios for VTI rocks. Obviously ν_{HV} can be higher than 0.5 (high limit of isotropic medium) because the lateral stronger resistance to deformation will be compensated in vertical direction. If VTI medium is infinite hard in horizontal direction comparing to vertical direction, then $\nu_{HV} \to 1$.

From eqns. (5) , (8) and $\nu_{HV} > 0$ we have

$$c_{11}c_{33} - {c_{13}}^2 > 0 \tag{10}$$

From eqns. (6) , (10) and $\nu_{HH} > 0$ we have

$$c_{13} < \sqrt{c_{33}(c_{11} - 2c_{66})} \tag{11}$$

From eqns. (5), (6) , (10) and $\nu_{HH} < \nu_{HV}$, we have

$$c_{13} > \sqrt{c_{33}(c_{11} - 2c_{66}) + {c_{66}}^2} - c_{66} \tag{12}$$

Combing eqn. (11) and (12), we put the constraints on c_{13} for VTI rocks in a neat form:

$$\sqrt{c_{33}c_{12} + {c_{66}}^2} - c_{66} < c_{13} < \sqrt{c_{33}c_{12}} \tag{13}$$

When VTI properties are reduces to isotropy ($c_{11} \to c_{33}$ and $c_{66} \to c_{44}$), the above inequalities reduce to

$$c_{13} = c_{33} - 2c_{44} \text{ and } K - \frac{2}{3}\mu > 0 \tag{14}$$

which agree with the physical limits for isotropic rocks.

Thomsen(1986) parameter δ is defined as:

$$\delta = \frac{(c_{13} + c_{44})^2 - (c_{33} - c_{44})^2}{2c_{33}(c_{33} - c_{44})} \tag{15}$$

From the definition δ monotonically increases with c13 when c_{13}>-c_{44}, so substituting the inequalities (13), into eqn. (15) and using Thomsen's (1986) notation, we can get the constraints for δ

$$\delta^- < \delta < \delta^+ \tag{17}$$

where

$$\delta^- = \frac{\varepsilon - 2{r_0}^2\gamma\left(1 - {r_0}^2(1 + 2\gamma) + \sqrt{\left(1 - {r_0}^2(1 + 2\gamma)\right)^2 + 2\varepsilon}\right)}{1 - {r_0}^2}$$

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$$\delta^+ = \frac{\varepsilon - 2{r_0}^2\gamma + {r_0}^2\sqrt{1 - 2{r_0}^2(1+2\gamma) + 2\varepsilon}}{1 - {r_0}^2}$$

where $r_0=\beta_0/\alpha_0$. So that δ is constrained by other Thomsen parameters, which are all properties in non-diagonal directions.

Lab data and the constraints

Fig. 4 shows crossplot between δ and ν_{HH}/ν_{HV} ratio from dynamic velocity anisotropy measurement. The data source are from Thomsen, 1986; Johnston and Christensen, 1995; Vernick and Liu, 1997; Jakobsen and Johansen, 2000; Wang, 2002; and Sone, 2012. If there are pressure dependent measurement, no more three data points are used for the same sample to prevent overweighting effect of this sample. The cross plot is divided into three areas. In the left, several data points have negative ν_{HH} values, the corresponding c_{13} are above the high bound and they tend to have higher values of δ. In the right area, there are quite a few points with $\nu_{HH} > \nu_{HV}$, the corresponding c_{13} values are lower than the low bound, and they tend to have lower values of δ. About 2/3 of the data points lie in the center area, where we believe that all true VTI media should lie within. Next we will analyzed that most of data points lying outside of bounds are due to uncertainty in lab velocity anisotropy measurement.

Uncertainty in lab velocity anisotropy measurement

Lab velocity anisotropy measurement on VTI media requires at least five velocity component measurements, among which one velocity measurement must be made in diagonal direction. Traditionally this diagonal velocity measurement is made on 45^o degree plug. Taking exact 45^o plug is difficult in practice, but people often ignore this angle error because the formula to calculate c_{13} is must simpler. As Yan et. al. (2012) pointed out, this small angle error can have significant effect on resulting c_{13} and δ. In Fig. 5, we take only data points satisfying $0 < \nu_{HH} < \nu_{HV}$, and assume true VTI properties are measured. Then taking the true phase velocity at phase angles 43^o, 40^o and 50^o respectively as phase velocity at phase angle 45^o, we recalculate c_{13}, and normalized c_{13} as follows:

$$c_{13n} = \frac{c_{13} - c_{13}^-}{c_{13}^+ - c_{13}^-} \tag{18}$$

where

$$c_{13}^- = \sqrt{c_{33}(c_{11} - 2c_{66}) + {c_{66}}^2} - c_{66}$$

$$c_{13}^+ = \sqrt{c_{33}(c_{11} - 2c_{66})}$$

As we can see from Fig. 5, negative 2^o angle error can make about 20% of the data points lie below the low bound; negative 5^o angle error can make about 62% of the

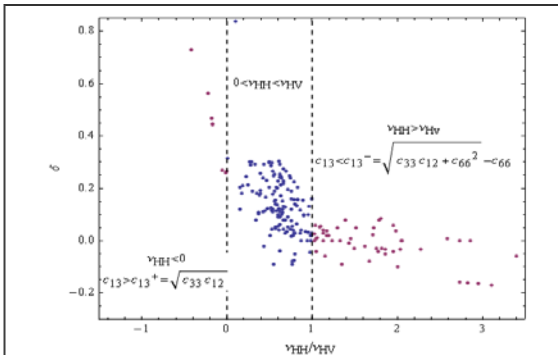

Figure 4: Crossplot between δ and ν_{HH}/ν_{HV} ratio from dynamic velocity anisotropy measurement (137 of 203 data points are within the bounds)

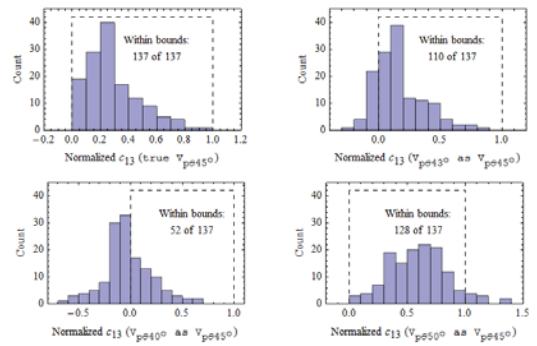

Figure 5: Effect of phase angle error on c_{13}

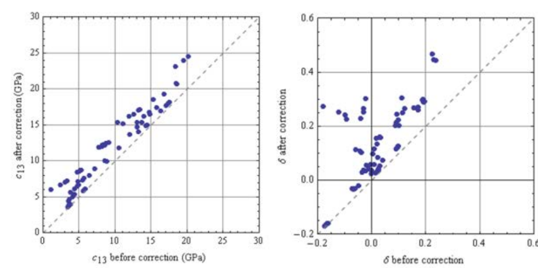

Figure 6: Group-phase correction on Wang's data (2002b, shale and coal samples only)

data points lie below the low bound; and positive 5^o angle error make about less than 8% of the data points lie above the high bound.

Another important issue is the group-phase problem. Dellinger and Vernik(1994) discussed related problems involved in traditional triple-plug velocity anisotropy measurement. To improve measurement efficiency, Wang(2002a) brought up a setup based on single horizontal plug. Using wavefront modeling, Yan et. al.(2013) analyzed that the diagonal velocity measured is actually

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group velocity. To calculate c_{13} and δ, we need to convert the group velocity to phase velocity and find the corresponding phase angle. Fig. 6 shows group to phase correction effect on resulting c_{13} and δ. It can be seen that if group velocity is mistook as phase velocity, c_{13} and δ will be systematically underestimated.

The above analysis explains that why there are more data points lying below the physical constraints of c_{13} and δ than above the constraints. Some other possible uncertainty (less likely) might come from heterogeneity of the sample, or the sample has fractures crossing the bedding(causing more horizontal deformation than in vertical direction under horizontal axial compression), in which case the sample does not really belong to VTI medium.

Prediction of δ

The physical constraints will help us understand the effect of other Thomsen parameters on δ. As shown in Fig. 7, when ε is constant, δ will increase with decreasing γ; when γ is constant, δ will increase ε. If ε is approximatly equal to γ, then δ will generally increase with degree of anisotropy. Small δ occurs when γ is much bigger then ε, even the anisotropy is strong. δ is less sensitive to raito of β_0/α_0 than other Thomsen parameters. For display convenience, we assume constant β_0/α_0 ratio 0.55, then plot the measured data with bounds of δ. As shown in Fig. 8, the trends of the approximated bounds comply well the the lab measured data.

Since δ is constrainted by non-diagonal properties, it might be possible that we can approximately predict δ without diaognal velocity measurement. In left panel of Fig. 9, using data points within the bounds, we directy correlate δ with other Thomsen parameters; in the right panel, we use the bounds of δ (eqn. (17)) to predict δ. Considering there are a lot of data points lying out of bounds, it is reasonable to believe the data points within the bounds should also have big uncertainty, thus the prediction results are encouraging. Also it should be noted that the samples coming from all over the world and are in different saturation and pressure conditions.

Conclusions

Laboratory velocity anisotropy measurement is challenging and there exists big uncertainty, which might obstruct our understanding of Thomsen parameter δ. The physical constraints should be useful for checking the data quality of velocity anisotropy measurement and understanding the relation between δ and other Thomsen parameters.

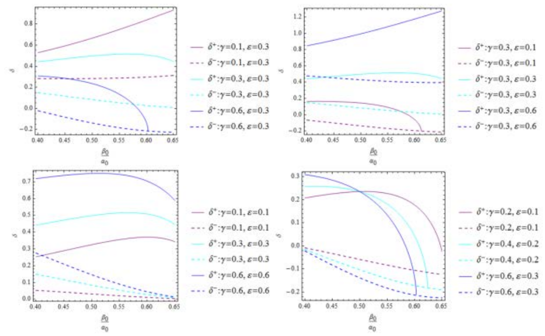

Figure 7 Relation between δ constraints and other Thomsen parameters.(If up bound curve crosses with low bound and terminates, it is because the up bound become complex number and some other physical relation is violated.)

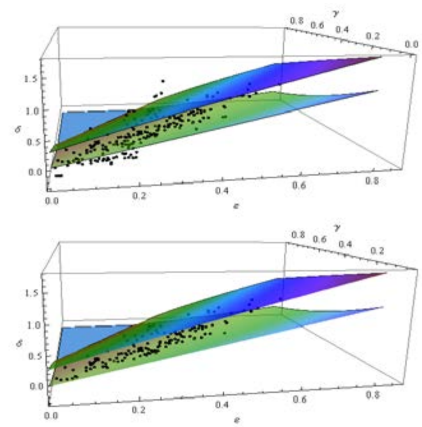

Figure 8: Comparison between measured and modeled ultrasonic velocities(above, all data; below, red data points in Fig. 4 removed).

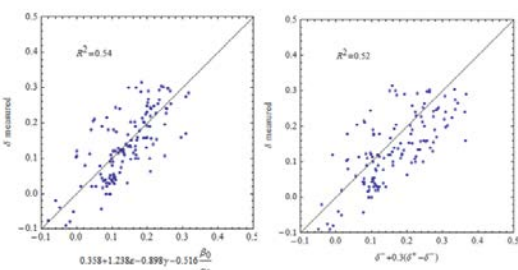

Figure 9: Prediction of δ from other Thomsen parameters

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EDITED REFERENCES

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