

## Limits of frequency effect on seismic anisotropy of sedimentary strata

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### Summary

The layer-induced seismic anisotropy of sedimentary strata is frequency dependent. At the low-frequency limit, the effective anisotropic properties of the layered media are estimated by the Backus averaging model. At the high-frequency limit, the apparent anisotropic properties of the layered media can be estimated by ray theory. Based on Monte Carlo simulation of the layered media using the laboratory ultrasonic-measurement database, the layering effects on seismic anisotropy at the high-frequency limit are compared with those at the low-frequency limit. Relative to the Backus averaging, ray theory usually underestimates the Thomsen parameters  $\epsilon$  and  $\gamma$  and overestimates  $\delta$ . For an effective layered medium consisting of isotropic sedimentary rocks, the differences are very significant. These differences decrease when shales with intrinsic anisotropy are included.

### Introduction

Sedimentary strata consist of layered sedimentary rocks. Observed from the outcrops and seismic profiles, the most prominent feature of sedimentary basins is often its layered structure. The elastic properties of a layered medium can be described by transverse isotropy (TI) and the properties are frequency dependent (Postma, 1955; Anderson, 1961; Backus, 1962; Helbig, 1984). The anisotropic properties of the subsurface sedimentary rocks can be determined from field seismic data, acoustic logging, and laboratory ultrasonic measurement. These data are acquired at different frequency ranges and scales. Knowing the frequency effect on seismic anisotropy is crucial for integration of different types of measurement data.

The low-frequency limit and high-frequency limit of seismic velocities in the TI symmetry axis are represented by the Backus averaging and ray theory, respectively. Silva and Stovas (2006) compared the equivalent anisotropic properties of an isotropic layer-cake model estimated by the Backus averaging and ray theory. The layer-cake model consists of two isotropic layers with given elastic properties. The model parameters are arbitrarily given and may be not very relevant to real sedimentary strata. The vertical property variations of a sedimentary formation are usually more complicated than a two-layer-cake model and some sedimentary layers may be intrinsically anisotropic. In this study, laboratory data based Monte Carlo simulation will be

applied to simulate more realistic sedimentary strata variations. Then the Backus averaging and ray theory will be applied to observe the differences in the anisotropic properties.

### Backus averaging and ray theory based velocity analysis in layered TI media

At the low frequency limit when the wavelength is much greater than the layer thickness, the effective properties of a layered medium composed of either isotropic or transversely isotropic layers can be estimated by Backus averaging (Backus, 1962).

At the high-frequency limit, the apparent seismic properties of a layered medium are transversely isotropic because seismic velocities are azimuthally independent in the bedding plane and only dependent on the incident angle. The measured properties are apparent because the layered medium is heterogeneous relative to the wavelength at high frequency limit. For a set of either isotropic or transversely isotropic layers, at the upper plane of the top layer, the reflection moveout from the lower plane of the bottom layer can be approximated by (Tsvankin and Thomsen, 1994; Tsvankin, 2012).

$$t^2(x) = t_0^2 + \frac{x^2}{V_{Pnmo}^2} - \frac{(V_{Phor}^2 - V_{Pnmo}^2)x^4}{V_{Pnmo}^2(t_0^2 V_{Pnmo}^4 + V_{Phor}^2 x^2)}. \quad (1)$$

If we know the total thickness of the layers, then the vertical velocity can be computed, and the apparent Thomsen parameters  $\delta$  and  $\epsilon$  can be determined using relations (Thomsen, 1986; Tsvankin, 2012).

$$V_{Pnmo} = V_{P0}\sqrt{1 + 2\delta}, \quad (2)$$

$$V_{Phor} = V_{P0}\sqrt{1 + 2\epsilon}. \quad (3)$$

For the SH-wave, the reflection moveout from the lower plane of the bottom layer is hyperbolic and is defined by

$$t^2(x) = t_0^2 + \frac{x^2}{V_{SHnmo}^2}. \quad (4)$$

If we know the vertical SH-wave velocity, the apparent Thomsen parameters  $\gamma$  can be determined using relation (Thomsen, 1986; Tsvankin, 2012),

$$V_{SHnmo} = V_{SH90} = V_{SH0}\sqrt{(1 + 2\gamma)}. \quad (5)$$

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Therefore, all the anisotropy parameters and TI elastic constants can be inverted from velocity analysis on the synthetic-seismic data produced from the layer-cake model.

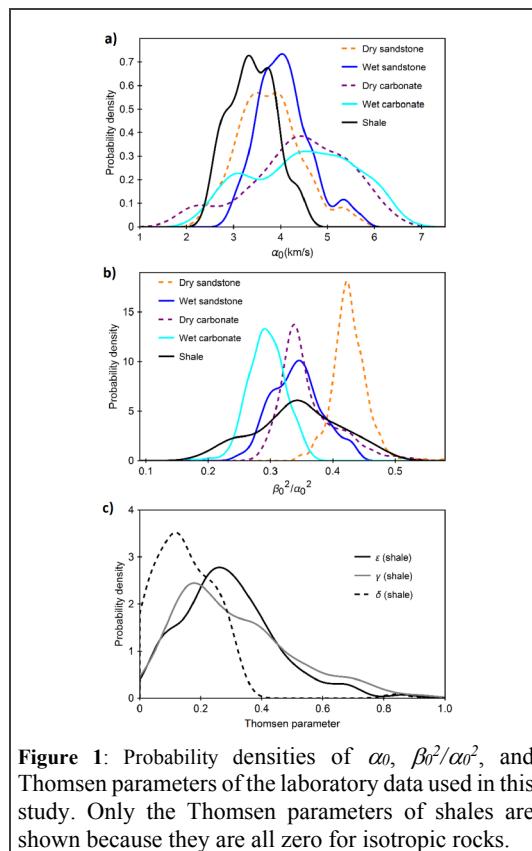
### Monte Carlo Simulation of Sedimentary Strata

The simulation is based on a large amount of laboratory ultrasonic measurement data of various sedimentary rocks. The sandstone data come from Han (1986). The carbonate rocks come from Rafavich et al. (1984), Kenter et al. (1997), Woodside et al. (1998), Assefa et al. (2003), and Fabricius et al. (2008). The shale data come from Thomsen (1986), Johnston and Christensen (1995), Vernik and Liu (1997), Jakobsen and Johansen (2000), Wang (2002), and Sone (2012). Detailed description of the data sources can be found in the studies by Yan et al. (2016a).

In our study, sedimentary strata are simulated by first randomly selecting a certain number of samples from a classification of rocks or a combination of classifications of rocks, and then the experimental data of the selected samples are used to parameterize the layer cake model. Considering the size of the database and the number of simulations, a 15-layer cake model is used. Once the model is parameterized, Backus averaging is directly used to determine the effective anisotropy properties of the layered medium. Based on the same model, Snell's law in anisotropic media is used to perform the ray tracing to calculate the reflection move-outs from the bottom layer. From the reflection move-outs, using the anisotropic velocity analysis technique introduced in the previous section, the apparent anisotropic properties of the layer medium can be estimated (Yan et al., 2016b). 1000 simulations are run for each category of sedimentary strata.

### Frequency limits of the layering effect on Thomsen parameters

Due to mild depositional environment changes, for the same type of rocks, the mineral composition and texture variation may also cause noticeable acoustic velocity change. The layering effect on seismic anisotropy may be different for different classifications of rocks. Fig. 2 shows the comparisons of the layering effects on seismic anisotropy at the low-frequency limit and high-frequency limit for each classification of rocks. In this study, subscript "B" denotes effective property by Backus average; subscript "R" denotes apparent property by ray theory. Compared to the Backus averaging, the P-wave anisotropy parameter  $\varepsilon$  is slightly underestimated for the shale layers, and it is significantly underestimated for the sedimentary strata consisting of isotropic layers. For the sandstone and wet carbonate layers,  $\varepsilon$  estimated by ray theory is less than half of that estimated by the Backus averaging; for the dry carbonate layers,  $\varepsilon$  estimated by ray theory is less than 1/3 of that estimated by the Backus averaging. Compared to the estimation of  $\varepsilon$ , the



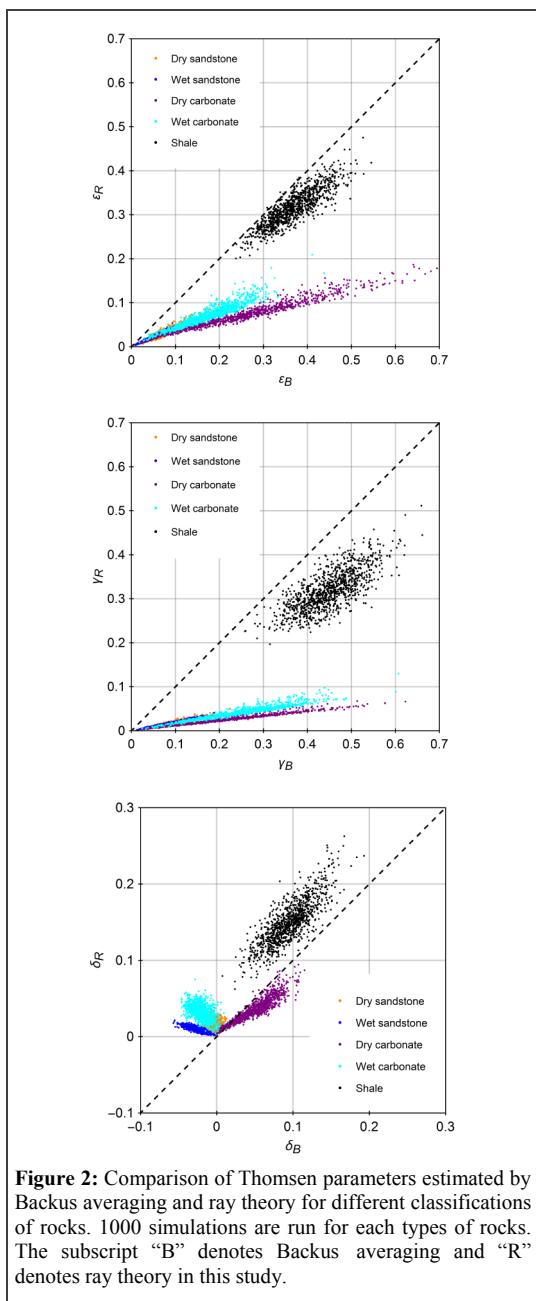
**Figure 1:** Probability densities of  $\alpha_0$ ,  $\beta_0^2/\alpha_0^2$ , and Thomsen parameters of the laboratory data used in this study. Only the Thomsen parameters of shales are shown because they are all zero for isotropic rocks.

S-wave anisotropy parameter  $\gamma$  is more significantly underestimated by ray theory relative to the Backus averaging.

For  $\varepsilon$  and  $\gamma$ , the ratio between the anisotropy parameters estimated by the Backus averaging and ray theory has little variation for the same classification of sedimentary rocks. Considering the compaction and diagenesis trend, random sampling from the experimental database may overestimates the elastic property variation of sedimentary strata. The high  $\varepsilon$  or  $\gamma$  values for some simulations may rarely happen in the real subsurface conditions. For the real sedimentary strata consisting of isotropic sedimentary rocks, the magnitude of  $\varepsilon$  or  $\gamma$  of may be much less than the simulation results, but the ratio between the anisotropy parameters estimated by the Backus averaging and ray theory should be similar the simulation results. Therefore, the simulation study still provide meaningful information.

The bottom panel of Fig. 2 shows a comparison of estimating  $\delta$  by the Backus averaging and ray theory. Except for the dry carbonate layers, the  $\delta$  values estimated by ray theory

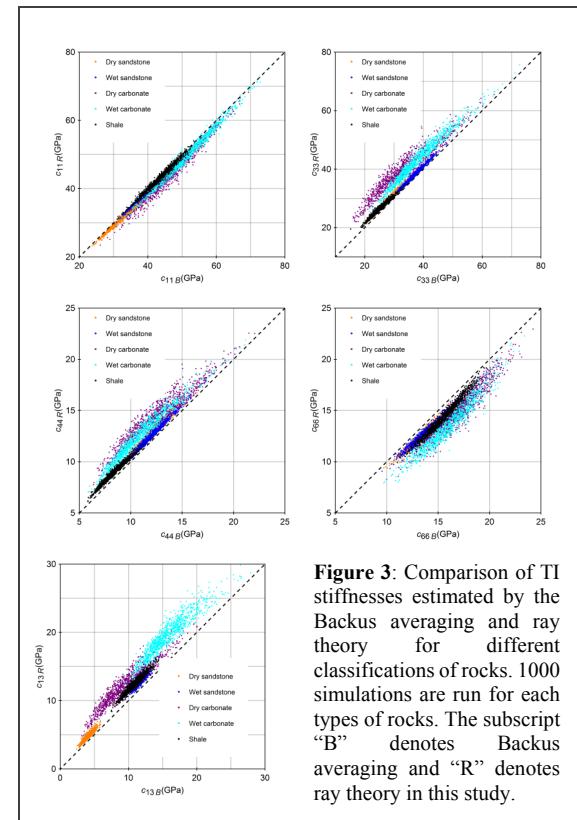
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are generally greater than those estimated by the Backus averaging. The  $\delta$  values estimated by Backus averaging for wet sandstone layers and wet carbonate layers are usually negative, but it is shown that the  $\delta$  values estimated by ray theory are always positive.

## Frequency limits of the layering effect on the TI Elastic constants

To have better understanding of the differences of the Thomsen parameters estimated by the Backus averaging and ray theory, it may be more instructive to compare the differences in the estimated TI elastic constants by the Backus averaging and ray theory. Fig. 3 shows the comparisons of the estimated TI elastic constants by the Backus averaging and ray theory. It can be seen that  $c_{11}$  estimated by ray theory is in general slightly less than estimated by the Backus averaging. The  $c_{33}$  estimated by ray theory is always higher than that estimated by the Backus averaging because  $c_{33}$  estimated by the Backus averaging is the Reuss low bound. The difference is determined by the difference between the Wyllie time-average and the Reuss bound. For sandstone and shale layers, the difference in estimated  $c_{33}$  values is small; the difference is more noticeable for the carbonate layers. According to the definition of the Thomsen parameter,  $\epsilon = (c_{11} - c_{33})/(2c_{33})$ , underestimating of  $c_{11}$ , and overestimating of  $c_{33}$ , both cause the estimated  $\epsilon$  by ray theory to be much smaller relative to that estimated by the Backus averaging. The case of estimating  $c_{44}$  is similar to that of estimating  $c_{33}$ . The difference in estimating  $c_{66}$  is much greater than the difference in estimating  $c_{11}$  by the two different approaches.



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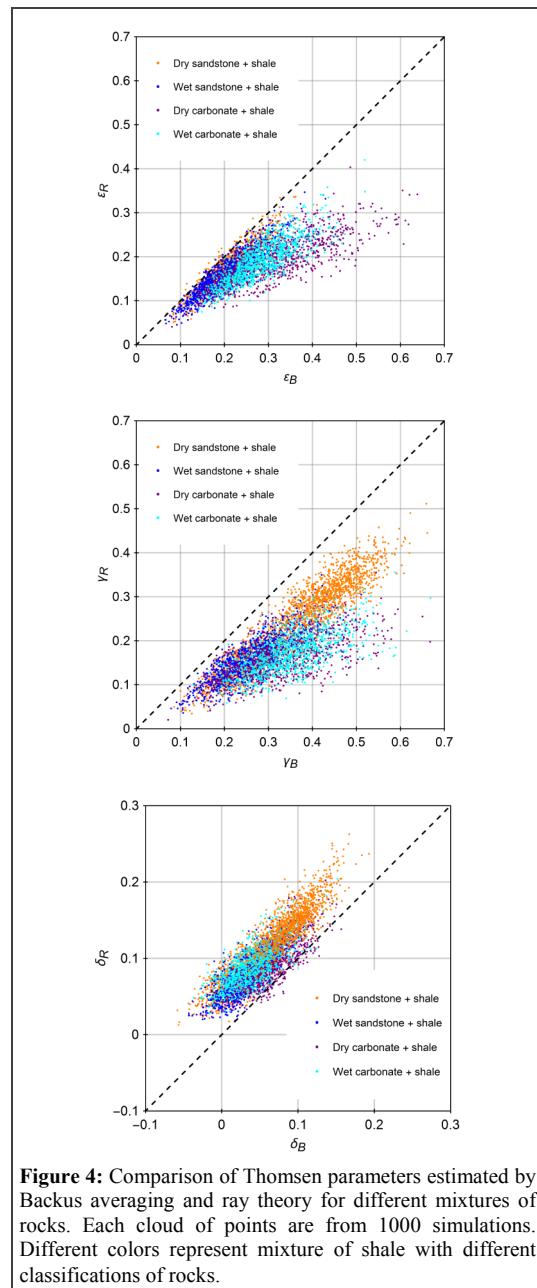
One possible explanation is that  $c_{66}$  estimated by Backus averaging is the Voigt high bound; whereas  $c_{11}$  estimated by the Backus averaging is less than the Voigt upper bound. Relative to the Backus averaging,  $c_{66}$  is significantly underestimated and  $c_{44}$  is overestimated by ray theory; therefore, the S-wave anisotropy parameter  $\gamma$  estimated by ray theory is significantly lower than estimated by the Backus averaging. The  $c_{13}$  estimated by ray theory is generally greater than estimated by the Backus averaging.  $\delta$  is most sensitive to  $c_{13}$  (Yan et al. 2012, 2016c). This is consistent with the fact that  $\delta$  estimated by ray theory is generally greater than that estimated by the Backus averaging.

### Frequency limits of the interbedding effect on seismic anisotropy

One more important and realistic factor for causing seismic anisotropy may be the interbedding of shales with sandstones or carbonate rocks. To model the difference of the interbedding effect on seismic anisotropy at the low-frequency limit and high-frequency limit, the sedimentary strata is modeled by randomly selecting statistically half of the layers parameterized by the experimental database of the shale samples and the other layers parameterized by the experimental database of one classification of the isotropic rock samples. Fig 4 shows the comparisons of Thomsen parameters estimated by the Backus averaging and ray theory for different mixtures of rocks. Comparing Fig. 4 with Fig. 2, the difference in estimating  $\epsilon$  of the interbedding formation is less drastic than the case of the sedimentary strata consisting of isotropic layers. Relative to the Backus averaging, underestimation of  $\epsilon$  by ray theory generally increases with  $\epsilon$ . The difference in estimating  $\epsilon$  is more significant for the interbedding of shales with carbonate rocks than the interbedding of shales with sandstones.  $\delta$  is generally overestimated by ray theory and the overestimation trend is approximately parallel to the perfect match. Comparing Fig. 2 with Fig 4, we can see that the thin beds of shales with intrinsic anisotropy are more important in determining the value of  $\delta$ .

### Conclusions

Based on Monte Carlo simulation of the sedimentary strata, the layering effects on seismic anisotropy at the low-frequency limit and at the high-frequency limit are compared. It is found that  $\epsilon$  and  $\gamma$  estimated by ray theory are usually less than those estimated by the Backus averaging and  $\delta$  estimated by ray theory is usually greater than estimated by the Backus average. For the sedimentary strata consisting of isotropic layers, the difference in the estimated  $\epsilon$  and  $\gamma$  by ray theory and the Backus average are very significant. The difference is less significant when the



**Figure 4:** Comparison of Thomsen parameters estimated by Backus averaging and ray theory for different mixtures of rocks. Each cloud of points are from 1000 simulations. Different colors represent mixture of shale with different classifications of rocks.

layer-cake model includes shale layers with intrinsic anisotropy.

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## EDITED REFERENCES

Note: This reference list is a copyedited version of the reference list submitted by the author. Reference lists for the 2017 SEG Technical Program Expanded Abstracts have been copyedited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

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