

## Rock physics constrained seismic anisotropy parameter inversion: A synthetic study

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### Summary

Previous studies have shown that it is very challenging to estimate the anisotropy parameters from seismic data even with a simple geological setting and very low noise level. The anisotropy parameters are not independent variables for organic mudrocks, whose elastic properties are often approximated by transverse isotropy. The relationships between the anisotropy parameters are usually not considered in the inversion process. We first modified a commonly used non-hyperbolic reflection moveout equation as a function of the interval anisotropy velocities so that rock physics constraints can be conveniently applied for each layer. The rocks physics constraints are constructed based on data analysis of selected laboratory anisotropy measurement data. The laboratory data are then used to parameterize hundreds of 15-layer transverse isotropy models by Monte Carlo simulation. The synthetic model testing shows that the accuracy of the estimated anisotropy parameters can be greatly improved if the relationships between the anisotropy parameters are considered during the inversion process.

### Introduction

Building a proper velocity model, which is critical for good quality seismic imaging, is not an easy task for the case of isotropic media. For the case of transverse isotropy (TI), at least two extra parameters for each layer are to be estimated from seismic data. This may lead to significant uncertainty in the estimated parameters. Grechka and Tsvankin (1998) made a feasibility study on the non-hyperbolic reflection moveout inversion. It is pointed out that the vertical velocity must be known for possible inversion of the anisotropy parameters, and the inverted value of  $\eta$  is highly sensitive to small correlated errors in the reflection travel times. Based on Monte Carlo simulation of a 15-layer model, Yan and Han (2018) made a sensitivity and accuracy analysis on anisotropy parameter estimation. It is found that small random travel time errors, which are insignificant compared to possible noises present in the common field data, can make the estimation of the Thomsen parameters  $\delta$  and  $\varepsilon$  almost infeasible.

No matter which method is used to estimate the anisotropy parameters, it is intrinsically a process of optimization to minimize the difference between the model and data. The relationships between the to-be inverted anisotropy parameters are usually not considered in the optimization process. Although the anisotropy parameters are theoretically independent, there are

relationships of various degree among them for a certain type of sedimentary rocks under study (Horne, 2013; Yan et al., 2016a). If these relationships can be used to constrain the inversion process, the accuracy of the estimated anisotropy parameters should be improved. Based on Monte Carlo simulation and synthetic modeling, this study demonstrates the feasibility of improving the accuracy of the estimated anisotropy parameters using rock physics constraints constructed from the laboratory measurement data.

### Direct interval anisotropy parameters inversion

The methodology we used to invert the anisotropy parameters is based on the non-hyperbolic reflection moveout equation brought up by Tsvankin and Thomsen (1994) and the inverting procedure is well documented by Tsvankin (2012). The reflection moveout equation is written in the form of

$$t^2(x, i) = t_0^2(i) + \frac{x^2}{V_{\text{Pnmo}}^2(i)} - \frac{(V_{\text{Phor}}^2(i) - V_{\text{Pnmo}}^2(i))x^4}{V_{\text{Pnmo}}^2(i)(t_0^2(i)V_{\text{Pnmo}}^4(i) + V_{\text{Phor}}^2(i)x^2)}. \quad (1)$$

where  $t$  is the two-wave travel time,  $t_0$  is the two travel time in the vertical direction,  $x$  is the source-receiver offset, and  $V_{\text{Pnmo}}$  and  $V_{\text{Phor}}$  are the stacking normal moveout velocity and horizontal velocity, respectively. The normal text ( $i$ ) denotes that the property is affected by all the overburden layers. The rock physics constraints can not be directly applied on the above equation because the stacking velocities are a type of apparent or pseudo properties, but the rock physics constraints are based on analysis of the effective properties. In order to apply the rock physic constraints, we need to revise the anisotropy parameters inversion procedure documented by Tsvankin (2012). From equation (7.25) by Tsvankin (2012), we get

$$V_{\text{Pnmo}}^2(i) = \frac{V_{\text{Pnmo},i}^2 t_{0,i} + V_{\text{Pnmo}}^2(i-1)t_0(i-1)}{t_0(i)}. \quad (2)$$

For the sake of simplicity, we follow Grechka and Tsvank's (1998) notation to use the "i" in the subscript to denote the interval attributes of the i-th layer.

From equation (7.23) by Tsvankin (2012), we can have

$$g(i) = \frac{V_{\text{Pnmo},i}^2(4V_{\text{Phor},i}^2 - 3V_{\text{Pnmo},i}^2)t_{0,i} + g(i-1)t_0(i-1)}{t_0(i)} \quad (3)$$

## Seismic Anisotropy Parameter Estimation

If we define

$$g_i = V_{\text{Pnmo},i}^2(4V_{\text{Phor},i}^2 - 3V_{\text{Pnmo},i}^2), \quad (4)$$

then, equation (3) can be written as

$$g(i) = \frac{g_i t_{0,i} + g(i-1)t_0(i-1)}{t_0(i)}. \quad (5)$$

From equation (7.21) in Tsvankin (2012), we have

$$V_{\text{Phor}}^2(i) = \frac{g(i)}{4V_{\text{Pnmo}}^2(i)} + \frac{3}{4}V_{\text{Pnmo}}^2(i). \quad (6)$$

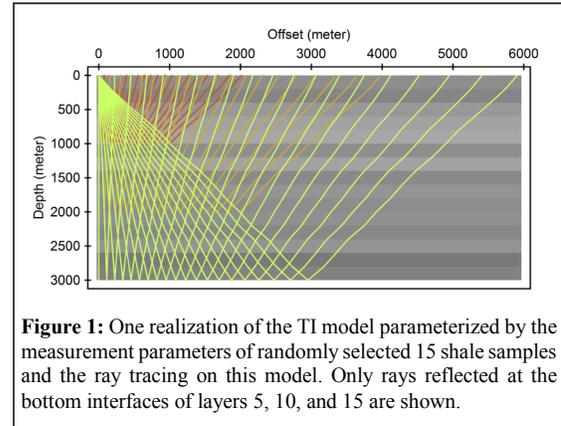
Substituting equations (2) and (5) into (6), we get

$$V_{\text{Phor}}^2(i) = \frac{g_i t_{0,i} + g(i-1)t_0(i-1)}{4(V_{\text{Pnmo},i}^2 t_{0,i} + V_{\text{Pnmo}}^2(i-1)t_0(i-1))} + \frac{3V_{\text{Pnmo},i}^2 t_{0,i} + V_{\text{Pnmo}}^2(i-1)t_0(i-1)}{4t_0(i)}. \quad (7)$$

Finally, substituting equations (2) and (7) into equation (1), we get a non-hyperbolic reflection moveout equation as a function of the interval velocities  $V_{\text{Pnmo},i}$  and  $V_{\text{Phor},i}$ . This equation is too clumsy to be put here. The solution is sought from the top layer to the bottom layer. When the interval velocities for the  $i$ -th layer are being sought, the stacking velocities at the bottom interface of the  $(i-1)$ -th layer are known properties that can be calculated from the interval velocities of the overlaid layers. By this way, the rock physics constraints can be directly applied on the modified non-hyperbolic reflection moveout equation during the optimization process. After the interval velocities are estimated, the Thomsen parameters  $\delta$  and  $\varepsilon$  (Thomsen, 1986) can be calculated.

### The synthetic modeling procedure

The modified anisotropy parameter estimating procedure is tested on a 15-layer TI model parameterized by Monte Carlo simulation. For each simulation, 15 samples are randomly selected from an anisotropy measurement database to parameterize the 15 layers with a layer thickness of 200 meters each. The laboratory anisotropy database includes the measurement data by Johnston and Christensen (1995), Sone (2012) and Vernik (2016). The dataset by Vernik (2016) is based on strict quality checking of previous measurements (Vernik and Nur, 1992; Vernik and Liu, 1997). The general modeling procedure is similar to that used by Yan and Han (2018), but the data resources are slightly different. In order to form effective rock physics constraints, only the datasets with better quality control are selected in this study. Yan et al. (2016b, 2018) explained the uncertainties in laboratory anisotropy measurement, and that why some datasets have better quality control in estimating  $\delta$  or  $c_{13}$



**Figure 1:** One realization of the TI model parameterized by the measurement parameters of randomly selected 15 shale samples and the ray tracing on this model. Only rays reflected at the bottom interfaces of layers 5, 10, and 15 are shown.

than the others do. After the TI model is parameterized, anisotropic ray tracing is performed to calculate the theoretical reflection traveltimes at the bottom interface of each layer. The maximum offset to depth ratio is controlled at  $2 \pm 0.2$  for the bottom interface of the model. Finally, the modified anisotropy parameter estimating procedure described in the previous section is used to invert the anisotropy parameters for each layer. Yan and Han (2018) have shown the layering effecting on the estimating of the anisotropy parameters. In this study, only the inverted results for Layer 15 are shown. This process is repeated for each realization of the 15-layer TI model and about 150 simulations are conducted so that the results not specific to a certain type of organic mudrocks.

Figure 1 shows one realization of the 15-layer TI model and the ray tracing at the bottom interfaces of Layer 5, 10 and 15. The elastic properties of each layer come from the measured properties of an organic shale sample, and the different gray levels denote the degree of P-wave anisotropy. Thomsen parameter  $\varepsilon$  is in the range of 0.05 to 0.7 for the selected three datasets. Based on the ray tracing, the two way reflection moveouts for each interface can be calculated. A certain level of random noises may be added to the travel times to test its effect on the estimating of the anisotropy parameters.

### Unconstrained anisotropy parameter inversion

We first run the simulation to test the validity of the modified procedure for direct interval anisotropy parameters inversion. When there is no noise added to the theoretical two-way travel times, the anisotropy parameters can be perfectly recovered in general using the modified procedure. The results are similar to the accuracy test by Yan and Han (2018) when the original anisotropy parameter procedure documented by Tsvankin (2012) is used.

## Seismic Anisotropy Parameter Estimation

On practical applications, there are always various noises presented in field seismic data and there are uncertainties in the estimated vertical velocities. Next, we repeat the anisotropy parameters estimating procedure after random noises within  $\pm 1.5$  ms are added to the travel times, and a random error within  $\pm 2\%$  is added to the vertical interval velocity. Figure 2 shows the comparisons between the true anisotropy parameters and the inverted anisotropy parameters for Layer 15 after the noise and error are added. At a depth of 3000 m, the added noises or error should be insignificant in relative to the noises presented in the real data. As can be seen from Figure 2, these mild noises made the anisotropy parameter estimating results generally unreliable.

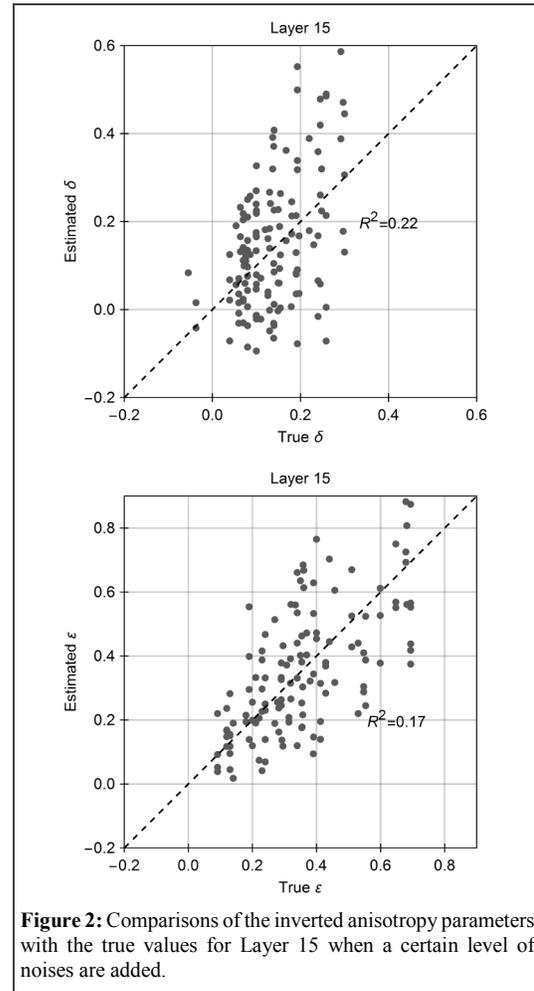
### Rocks physics constraints

Although the elastic properties of isotropic rocks are described by two theoretically independent parameters, for example, P-wave velocity and shear wave velocity, the two parameters are not practically independent for the sedimentary rocks. There are strong correlations between the P-wave velocity and shear wave velocity, and the  $V_p$ - $V_s$  ratio is often a very good lithology identifier. The elastic properties of organic mudrocks are often approximated by transverse isotropy. Similarly, the five anisotropy parameters describing the anisotropic properties of the organic mudrocks are not independent (Horne, 2013; Yan et al., 2016a). Studying the relationships between the anisotropy parameters is critically important for practical application of seismic anisotropy.

Figure 3 shows the various relationships between the velocity anisotropy parameters based on the measurement data by Johnston and Christensen (1995), Sone (2012) and Vernik (2016). For organic mudrocks, these velocities are more or less correlated. If we assume that what the laboratory data measured are effective properties and the upscaling from the core scale to the formation scale does not affect the relationships, then the inverted anisotropy parameters should also be consistent with these relationships. Next, the approximate boundaries of the data trends denoted by the dashed lines in panels (a), (b) and (c), and the boundaries of the 95% confidence area shown in panel (d) of Figure 3 will be used as the rock physics constraints for anisotropy parameter estimating.

### Constrained seismic anisotropy parameter inversion

For the testing in this section, the procedure and the noise level are same as those in the unconstrained anisotropy parameter inversion conducted in the previous section except that the rock physics constraints constructed in the last section are applied. The Monte Carlo simulation of



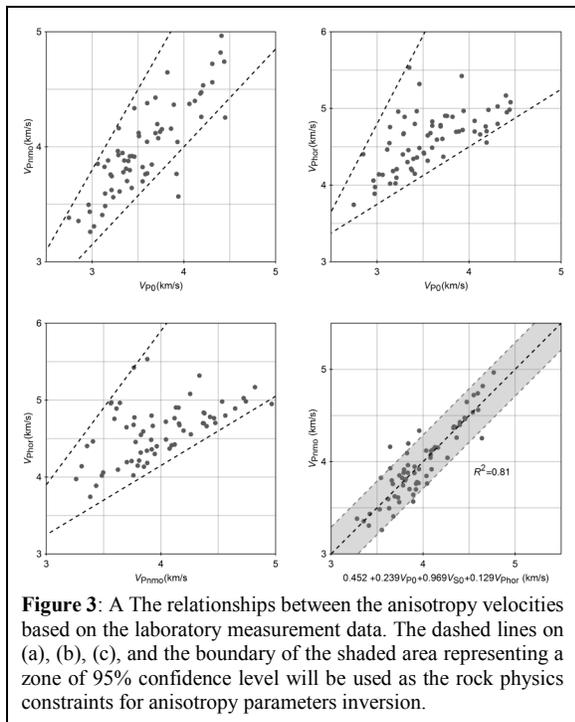
**Figure 2:** Comparisons of the inverted anisotropy parameters with the true values for Layer 15 when a certain level of noises are added.

the 15-layer TI model is based on the same data resources, and the vertical layer properties are assumed known as well. The constraints are applied on the optimization process to minimize the error between the reflection moveout equation that is modified as a function of the interval velocities and the two wave travel times at the bottom interface of the layer. The built-in tool “NonlinearModelFit” in Mathematica 11.2 is utilized to perform the constrained optimization.

Figure 4 shows the comparisons between the estimated Thomsen parameters  $\delta$  and  $\epsilon$  and the true layer properties for Layer 15 after the rock physics constrained are applied during the inversion process. Compared to Figure 2, there are obvious improvements in the accuracy of the estimated anisotropy parameters. When noises are present, there may be many combinations of the anisotropy parameters that can lead to similar

## Seismic Anisotropy Parameter Estimation

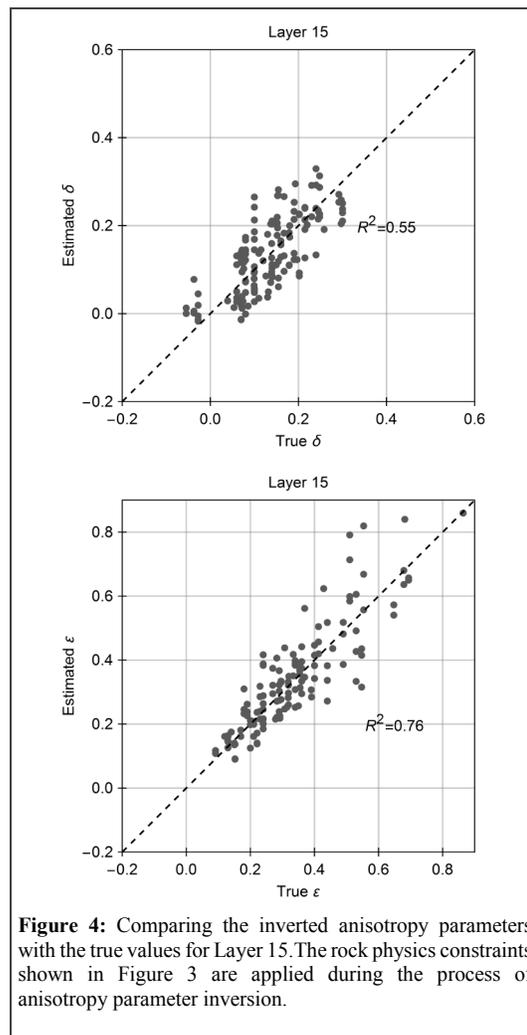
minimization of the object function, but some of the combinations may be way off from the relationships shown in the last section.



For the estimating of  $\delta$  or  $c_{13}$ , the measurement data by Johnston and Christensen (1995) and Sone (2012) have better data quality control than the other anisotropy measurement data because it is based on pure phase velocity measurement on multiple core plugs in the non-principal directions. From another point of view, the organic mudrocks are not necessarily perfect TI media, a more representative value of  $\delta$  or  $c_{13}$  can thus be approximated from multiple phase velocities in different directions. The Thomsen parameter  $\delta$  is primarily related to normal moveout velocity. The correlation of  $V_{Pnmo}$  with  $V_{P0}$  and  $V_{Phor}$  using the measurement data by Johnston and Christensen (1995) and Sone (2012) is stronger than the correlation shown in panel (d) of Figure 3 even with one less parameter is included. The stronger relationship between the anisotropy parameters should be more effective in improving the accuracy of the inverted anisotropy parameters.

### Conclusions

The non-hyperbolic reflection moveout equation is modified as a function of the interval velocities so that estimating the anisotropy parameters from seismic velocity analysis can be



effectively constrained by the rock physics relationships. The synthetic model testing shows that the accuracy of the estimated anisotropy parameters can be substantially improved if proper rock physics constraints are applied during the optimization process.

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