

## Gain Function and Hydrocarbon Indicators

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### Summary

We have derived the gain function for porous rock based on the Gassmann's equation. We study property of the gain function for consolidated sandstone and poor consolidated sands from deep-water, Gulf of Mexico. We apply the gain function to derive pore fluid modulus as a hydrocarbon indicator from log data.

### Introduction

Seismic detection of reservoir fluids is a continuous challenge in exploration. Fluid saturation effects on velocity depend on rock and fluid properties at in situ condition, and their interaction, which depends on wave frequencies (Batzle et, al. 2001). Although velocities can be measured directly, they are not the best fluid indicators.

$$V_p = \sqrt{\frac{K + 4/3\mu}{\rho}} \quad V_s = \sqrt{\frac{\mu}{\rho}} \quad (1)$$

where  $K$  is bulk modulus,  $\mu$  is shear modulus and  $\rho$  is density. P- and S-wave velocities are correlated to each other through shear modulus. Both P- and S-wave velocities also depend on density.

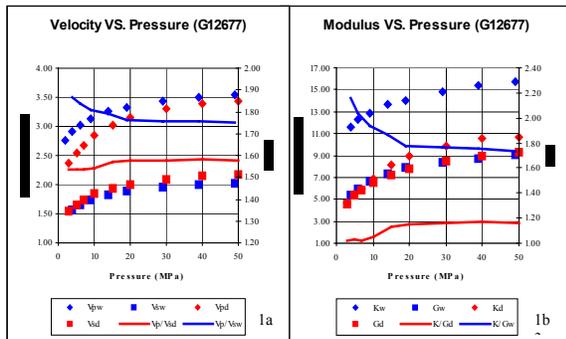


Figure 1a) and b) P- and S-wave velocities and bulk and shear moduli as function of pressure at dry and fully water saturated condition.

Measured data on a typical porous sandstone show that velocity (Figure.1a) is not sensitive to water saturation. P-wave velocity slightly increases, while S-wave velocity decreases. However, the bulk modulus is very sensitive to water saturation. It increases 50%, while shear modulus remains as a constant (Figure.1b). It is not surprising, because water has a high incompressibility, but negligible rigidity. A decrease of shear velocity is due to an increase of density with water

saturation. Clearly, bulk and shear moduli are the most important parameters to distinguish the fluid effects.

Gassmann's equation is usually used to describe the fluid saturation effects on seismic velocity. A general assumption is that pore fluid pressure generated in seismic wave front has enough time to equilibrium. In this study, we use the Gassmann's equation to define "Gain Function". Then, we discuss properties of the gain function for consolidated sandstone and weakly cemented sands from deep-water.

### The Gain Function

Gassmann's equation (1951) can be written as

$$K_s = K_d + \Delta K_d$$

$$\Delta K_d = \frac{K_0(1 - K_d / K_0)}{1 - \phi - K_d / K_0 + \phi K_0 / K_f}$$

and

$$\mu_s = \mu_d \quad (2)$$

where  $K_0$  is mineral modulus,  $K_f$  is fluid modulus,  $\phi$  is porosity. The fluid saturated bulk modulus  $K_s$  is equal to the dry bulk modulus  $K_d$  plus an increment of bulk modulus ( $\Delta K_d$ ) due to the fluid saturation effect, while shear modulus  $\mu$  remains as a constant. For high porosity sandstone (>15%),  $\Delta K_d$  can be derived (slightly overestimated) as (Han and Batzle, 2003).

$$\Delta K_d \approx G_{simp}(\phi) \times K_f = \frac{(1 - K_n)^2}{\phi} \times K_f \quad (3)$$

We have emphasized important of fluid properties all along (Batzle and Wang, 1992, Han and Batzle, 2000 a, b, Han and Batzle, 2002a). This study, we focus on the rock property. The simplified gain function  $G_{simp}(\phi)$  is property of dry rock frame: normalized modulus  $K_n (= K_d / K_0)$  and porosity. We expect the simplified gain function is not dispersive, same as the dry rock velocity (Spenser, 1981; Winkler, 1986). The gain function concept helps to understand the fluid saturation effect: the increment of bulk modulus approximately equal to the product of the fluid modulus and the gain function of the dry rock frame. Base on the above analysis, we extend definition of the gain function based on Gassmann's equation as:

$$G(\phi) = \Delta K_d / K_f \quad (4)$$

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where  $\Delta K_d$  is now the increment of bulk modulus from Gassmann's equation (not the simplified formulation). Notice that  $G(\phi)$  is approximately property of a dry frame and derived with low frequency limit.

### Properties of the gain function

We can establish the bounds of the gain function based on the Voigt and Reuss bounds.

For the Voigt bound, normalized modulus  $K_d/K_0$  equal to

$$K_{nV} = (1 - \phi) \Rightarrow G_V = \phi \quad (5)$$

The Voigt bound is the elastic maximum for porous rocks, which then has the minimum fluid saturation effect. The  $G_V$  is the minimum bound for the gain function.

For the Reuss bound,  $K_{nR} = 0$ , and the gain function equal to

$$G_R = \Delta K_R / K_f = \frac{K_0}{(1 - \phi)K_f + \phi K_0} \leq G_{R,simp} = \frac{1}{\phi} \quad (6)$$

The Reuss bound is the elastic minimum of porous rocks, which has the maximum fluid saturation sensitivity. The  $G_R$  is the high limit of the gain function. For a suite of fluid modulus of 3, 2, 1, 0.5 and 0.1 GPa and mineral bulk modulus  $K_0 = 40$  GPa, the gain function of the Reuss bound  $G_R$  increases with decreasing fluid modulus and porosity as shown in Figure.2. As the fluid modulus approaches zero, the gain function  $G_R$  approaches  $1/\phi$ , equivalent to the simplified gain function  $G_{R,simp}$ .

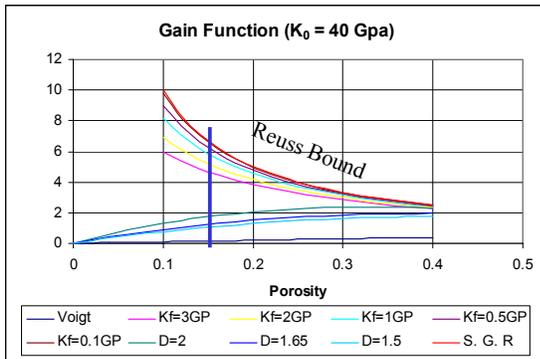


Figure 2. The gain function with different fluid modulus and the gain function based on D-function of sandstone and weakly cemented sands.

For reservoir rocks with high porosity, the simplified-gain function is slightly overestimation of the gain

function. And the fluid modulus effect can be neglected. However, this is not the case for rocks with low porosity (<15%). At low porosity, the gain function of the Reuss bound increases with decreasing fluid modulus, which are limited by  $1/\phi$  as shown in Figure 2.

### The simplified-gain function based on D-function

Here, we assume that the normalized modulus for dry sandstone at high pressure can be written as  $(1 - D \cdot \phi)^2$ , where D is a constant (Han and Batzle, 2003). This D parameter represents the degree of compaction and consolidation. It ranges from 1.45 to 2 for consolidated sandstones. Therefore, we can write the (simplified) gain function based on equation (3) as

$$G_{simp}(\phi) = D^2 \times \phi \times (2 - D \times \phi)^2 \quad (7)$$

Figure.2 shows the simplified-gain function for D of 2, 1.65 and 1.5, which represents the range of the D parameter for consolidated sands at high differential pressure. For clean sandstone, the simplified-gain function tends to increase with increasing porosity (<30%) and D parameter.

### Empirical Gain function

We have measured a suite of clean and shaly sandstone (Han, 1986). From measured dry velocity data we have calculated the gain function for the group of samples. Figure 3 shows the gain function as function of porosity. The data show two trends. For clean sandstones, the gain function increases with increasing porosity. For reservoir sands (porosity > 15%) the gain function is distributed in a narrow range (red ellipse). For low porosity rocks, the gain function shows a large scatter but is mainly controlled by clay content (brown circle). The gain function shows a linear increase with clay content (Figure.3).

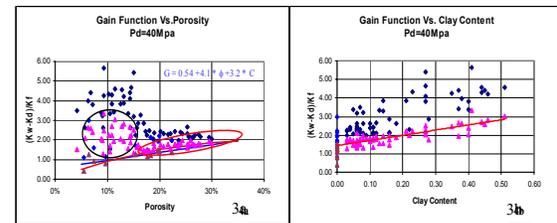


Figure.3. Calculated the gain function (pink) versus (a) porosity and (b) clay content based on Gassmann's equation in comparison with the data from water-saturated velocity in ultrasonic (blue).

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For consolidated sandstones at differential pressure of 40 MPa, we have derived the gain function as a linear function of porosity and clay content (blue line) from 31 reservoir sands with porosity more than 15% and clay content 10% or less.

$$G_{Gass}(\phi, C) = a + b * \phi = 0.54 + 4.1 * \phi \quad (8)$$

where a, b are regression parameters. Clay effect can be included with coefficient of 3.2. The correlation coefficient equals to 0.82. The gain function increases with increasing porosity and clay content. For clean sands, equation (8) provides a lower bound of the gain function as shown by the blue line in Figure 3a. We can also generate an empirical-gain function base on equation (7) with D parameter of 1.65:

$$G_{emp}(\phi) = 1.65^2 * \phi * (2 - 1.65 * \phi)^2 \quad (9)$$

The equation (9) fits porous sands well as shown by the red line in Figure.3. The D value needs to be calibrated locally.

The ‘gain’ function calculated from ultrasonic data (blue) in Figure 3 is consistent with low frequency data (based on Gassmann’s calculation) at high porosity (>25%). But it is significantly dispersed to higher for samples having low porosity and high clay content.

### Pressure effect

The gain function tends to increase with both decreasing cementation and differential pressure. Equation (9) with D = 1.65 is a lower bound of the gain function for consolidated clean sandstones at high pressure. The pressure effect on gain function is small at high pressures (50 to 30 MPa), but significant at lower pressure, especially for clay-rich or creak-rich samples.

### Gain function of Deep-water Sands

For loose and weakly cemented sands from deep-water, the Gulf of Mexico, the gain function is much higher than those of consolidated sand as shown in Figure 4. For high porosity sands (>30%) with little cementation, the gain function is almost a constant 2.5, which is less sensitive to both differential pressure and porosity. For sands with porosity less than 30% and weak cementation, the gain function decreases from 3.0 to around 2.5 with increasing differential pressure. The gain function model (equation 7) with a D parameter equal to 2.1 fits the data measured at high pressure well.

$$G(\phi) = 2.1^2 * \phi * (2 - 2.1 * \phi)^2 \quad (10)$$

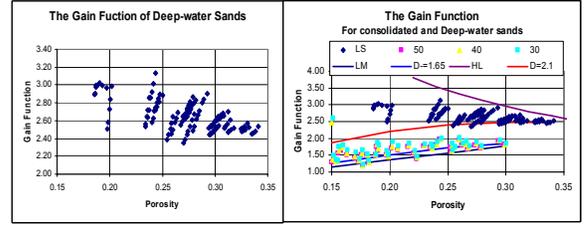


Figure 4. The gain function of weakly cemented sands from deep-water, the Gulf of Mexico in comparison with the gain function of porous consolidated sandstone.

The difference between equation (7) and (10) is an indication of the maximum cementation effect. The important issue raised by this study is that for porous rock, the gain function distributes in a narrow range. It is not sensitive to pressure (> 7 Mpa) and porosity (>30%). And it is predictable (around 2.5).

One of the primary applications for the gain function is to derive fluid modulus. We can use the gain function of reservoir sands to derive modulus of pore fluids.

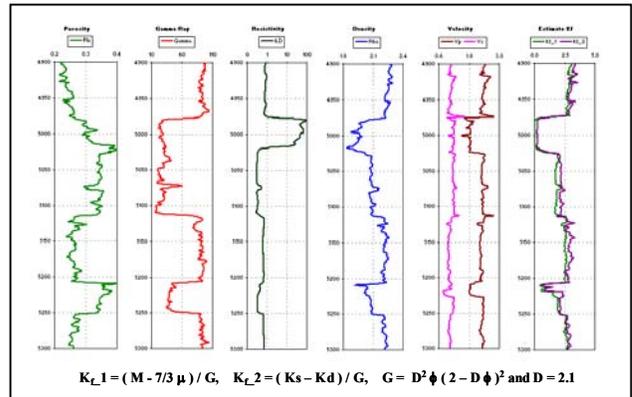


Figure 5. Predicted fluid modulus in a sand and shale sequence from a suite of log data of Gulf Mexico.

Figure. 5 is a suite of log data (Porosity, Gamma Ray, Resistivity, Density, P- and S-wave velocity) from the Gulf of Mexico. It is a typical shale-sand-shale reservoir. The top sand is 140 ft thick with gas saturation in the top most 50 ft. We have two ways to derive pore fluid modulus.

1.  $K_{f1} = (M - 7/3 \mu) / G$  (Batzle et.al, 2001)
2.  $K_{f2} = (Ks - Kd) / G \quad (11)$

We can use the gain function (equation 10) to derive the pore fluid modulus. The results show that the two methods

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generate similar results.  $K_{f,1}$  is slightly smaller than  $K_{f,2}$ . In water zone K is around 2.3 Gpa, which is typical for shallow brine with low salinity. In the gas zone, fluid modulus reduces to 0.2 GPa or less, and this is typical for gas-saturated reservoirs.

In shaly sands and shale zones the derived fluid modulus will be overestimated because the low gain function of porous sands is used. In shale zones, calculated fluid modulus is ranging from 2.5 to 3.0 Gpa, which suggests: high fluid modulus is a shaleness indicator.

### Conclusion

For porous deep-water sands from the Gulf Mexico, fluid saturation effect (increment of bulk modulus) can be simplified as the product of the gain function of sands and modulus of pore fluid. The gain function of porous sands at high pressure ( $P_d > 7$  Mpa) is narrowly distributed (around 2.5). Using the gain function we may be able to derive pore fluid modulus seismically.

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